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A CONTINUITY THEOREM AND SOME COUNTEREXAMPLES FOR THE THEORY OF--ETC(U)

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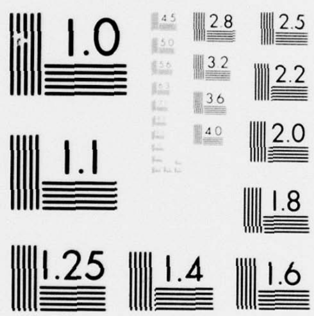

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A Continuity Theorem and Some Counterexamples for the Theory of Maintained Systems

by

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A CONTINUITY THEOREM AND SOME COUNTEREXAMPLES FOR THE THEORY OF MAINTAINED SYSTEMS[1]

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__Abstract.__ The reliability of maintained systems is considered. A "continuity theorem" is presented which states that the stochastic behavior of a maintained system depends continuously on the stochastic behavior of its components. Examples of maintained systems with IFR component lifetimes and exponential repair times are presented for which time until first system failure is not NBU.

__Key words and phrases:__ Reliability theory, maintained systems, time until first system failure, continuity theorem.

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Several authors [1,2,4,9,12,13] consider coherent systems in which some of the components are repairable. A general model is as follows [2]: Component positions 1 through k are filled with repairable components with lifetime distributions F_i and repair time distributions G_i, $i = 1,2,\ldots,k$. Components $k+1$ through $k+m$ are nonrepairable with lifetime distributions F_i, $i = k+1,\ldots,k+m$. Components function independently of each other, continuing to function while other components are being repaired. Barlow and Proschan (1976) show that the time until first system failure (starting with all new components) is NBU when the repair times are DFR and component lifetimes are exponential. They conjecture that this is also true for IFR component lifetimes. The purpose of this paper is to present counterexamples to this conjecture. Other examples will also show that increased repair rates do not necessarily result in greater system reliability or availability for repairable systems with IFR components. In the course of verifying these examples a "continuity theorem" for coherent maintained systems is presented, which in effect says that systems with the same structure function and approximately equal component distributions will have approximately the same system behavior.

1. Deterministic Examples (1-out-of-2 systems).

Consider a 1-out-of-2 system with independent component lifetimes and repair times. Following the notation of the introduction, let

$$\begin{aligned} F_1(t) &= 1_{[a_1,\infty)}(t) \\ G_1(t) &= 1 - \exp(-\lambda_1 t) \\ F_2(t) &= 1_{[a_2,\infty)}(t) \\ G_2(t) &= 1 - \exp(-\lambda_2 t) \end{aligned} \tag{1.1}$$

Note that both F_1 and F_2 are IFR as defined for arbitrary distributions by Barlow and Proschan(1975), p. 54. At time 0 the system consists of new components. Let H be the cdf of the time until first system failure, L. The *reliability of the system for the period* $[0,t]$ is $R(t) = 1 - H(t) = \overline{H}(t)$. The system is NBU if $\overline{H}(s+t) \leq \overline{H}(t)\overline{H}(s)$ for all $s,t \geq 0$. Availability of the system, $A(t)$, in $[0,t]$ equals the amount of time which the system functions during $[0,t]$; it is a random variable. $(P(A(t)=t) = \overline{H}(t).)$ We consider two special cases: the first is not NBU; in the second case the reliability decreases for certain increases in repair rate.

i) Let the constants in equation (1.1) be:

$$a_1 = 10 \qquad \lambda_1 = 1 \qquad a_2 = 9 \qquad \lambda_2 = 0.1 \tag{1.2}$$

Let R_1 (R_2) be the first repair time of the first (second) component, then

$$P(\ L > 11\ ) \ = \ P(\ R_2 \leq 1\ ) \ = \ 1 - \exp(-\lambda_2) \ = \ .095$$

and

$$\begin{aligned} P(\ L > 22 \mid L > 11\ ) \ &> \ P(\ 2 \leq R_1 \leq 8 \mid R_2 \leq 1\ ) \\ &= \ P(\ 2 \leq R_1 \leq 8\ ) \\ &= \ \exp(-2\lambda_1) - \exp(-8\lambda_1) \\ &= .135 \end{aligned}$$

Thus

$$P(\ L > 22 \mid L > 11\ ) = \overline{H}(22)/\overline{H}(11) \ > \ \overline{H}(11)$$

and the system is not NBU.

ii) Let the constants in equation (1.1) be:

$$a_1 = 2 \qquad \lambda_1 = \text{variable} \qquad a_2 = 4 \qquad \lambda_2 = 0.1 \tag{1.3}$$

Again letting L equal the time until first system failure and R_1 (R_2) equal the first repair time for the first (second) component, then the event $\{ L > 5 \}$ is equivalent to the event $\{ 1 \le R_1 \le 2 \} \cup \{ 0 \le R_1 < 1 , 0 \le R_2 \le R_1 \}$. Using the independence of R_1 and R_2 gives

$$P(L > 5) = 1 - \exp(-2\lambda_1) - \lambda_1(\lambda_1+\lambda_2)^{-1}(1-\exp(-\lambda_1-\lambda_2))$$

If $\lambda_2 = 0.1$ then the following table gives $P(L > 5)$ for various values of λ_1:

λ_1	0.5	0.6	0.7	0.8	0.9	1.0
$P(L > 5)$	.256	.267	.272	.271	.266	.258

Thus increasing λ_1 does not necessarily increase (stochastically) the time until first system failure. Nor does it necessarily increase availability because $P(A(5) = 5) = P(L \geq 5)$.

2. A Continuity Theorem for Maintained Systems.

The examples in the previous sections are quite special: their components have deterministic lifetimes. We desire examples with random lifetimes which are IFR; in particular, it would be nice to find an example with absolutely continuous lifetime distributions and furthermore with bounded failure rates. To verify the existence of such examples the following theorem is useful.

Continuity Theorem 2.1. A coherent system Σ_0 consists of k repairable components and m nonrepairable components with component lifetime distributions $F_{0,i}$, $i=1,2,\ldots,k+m$, and component repair time distributions $G_{0,i}$, $i=1,2,\ldots,k$, such that $F_{0,i}(0) = 0$ and $G_{0,i}(0) = 0$. At time 0 all components are new. Let L_0 be the time until first system failure and $A_0(t)$ be the system availability in $[0,t]$. Suppose that the system

Σ_n has the same structure function as Σ_0 with component lifetime and repair distributions $F_{n,i}$, $i=1,2,\ldots,k+m$, and $G_{n,i}$, $i=1,2,\ldots,k$. Suppose that $F_{n,i} \xrightarrow{D} F_{0,i}$ and $G_{n,i} \xrightarrow{D} G_{0,i}$ as $n \to \infty$ for all i. Let $A_n(t)$ equal the system availability in $[0,t]$ of Σ_n. Then $A_n(t) \xrightarrow{D} A_0(t)$ as $n \to \infty$. Let L_n be the time until first system failure of Σ_n. If different components of Σ_0 have simultaneous state-transitions with probability 0, then $L_n \xrightarrow{D} L_0$ as $n \to \infty$. " $\xrightarrow{D}$ " signifies convergence in distribution.

Proof: For integers $n \geq 0$, $1 \leq i \leq k$, $j \geq 1$, let $Q^j_{n,i}$ be independent lifetimes with cdf $F_{n,i}$ and $R^j_{n,i}$ independent component repair times with cdf $G_{n,i}$. Define

$$T^{2\ell}_{n,i} = \sum_{j=1}^{\ell} (Q^j_{n,i} + R^j_{n,i}) , \qquad T^{2\ell+1}_{n,i} = T^{2\ell}_{n,i} + Q^{\ell+1}_{n,i}$$

$\ell = 0,1,2,\ldots$. For each i,n, $\{T^j_{n,i}, j=0,1,2,\ldots\}$ is an alternating renewal process. Letting $n \to \infty$, the finite dimensional distributions of these sequences converge to those of $\{T^j_{0,i}, j=0,1,2,\ldots\}$, for each $i = 1,2,\ldots,k$, because the summands converge in distribution. The f.d.d.'s are convergence-determining for the product topology on R^∞ ([3], p.19) thus

$$\{T^j_{n,i}, j=0,1,2,\ldots\} \Rightarrow \{T^j_{0,i}, j=0,1,2,\ldots\}$$

in the product topology as $n \to \infty$ for $i=1,2,\ldots,k$. It follows from renewal theory that these processes have paths almost surely in

$$S = \{(t_0,t_1,\ldots) \in R^\infty: \ 0 = t_0 \leq t_1 \leq t_2 \leq \ldots , \lim_{j\to\infty} t_j = \infty\} .$$

In addition, because $F_{0,i}(0) = 0$ and $G_{0,i}(0) = 0$, $T_{0,i}$ has paths almost surely in

$$S_0 = \{(t_0, t_1, \dots) \in R^\infty: \ 0 = t_0 < t_1 < t_2 < \dots, \ \lim_{j\to\infty} t_j = \infty\} .$$

Let π denote the product topology on R^∞ and also its relativization to S. Let $\mu_{n,i}$ denote the measures induced on $(S, \sigma(\pi))$ by $T_{n,i}$.

Now consider another representation of the above alternating renewal process: for integers $n \geq 0$ and $1 \leq i \leq k$, define

$$X_{n,i}(t) = \begin{cases} 1, & T_{n,i}^{2k} \leq t < T_{n,i}^{2k+1} \\ 0, & T_{n,i}^{2k+1} \leq t < T_{n,i}^{2k+2} \end{cases}$$

$k=0,1,2,\dots$. Thus if the i^{th} component of Σ_n is functioning at time t, $X_{n,i}(t)=1$; if it is under repair, $X_{n,i}(t)=0$. The processes $X_{n,i}$ are called "operating processes." Because $\lim_{j\to\infty} T_{n,i}^j = \infty$ a.s., $X_{n,i}$ will be random functions in $D[0,\infty)$, a.s. In particular, let $B[0,\infty) \subset D[0,\infty)$ be the binary 0-1 functions, then $X_{n,i}$ takes values in $B[0,\infty)$. Endow $B[0,\infty)$ with a topology for which convergence is defined as follows: $x_n \to x$ if there exists a sequence of continuous one-to-one mappings λ_n of the interval $[0,\infty)$ onto itself such that $x_n = x \circ \lambda_n$ and for each $M > 0$ $\lim_{n\to\infty} \sup_{0\leq t\leq M} |\lambda_n(t) - t| = 0$. This topology is the relativization to $B[0,\infty)$ of Skorohod's J_1 topology defined by Stone (1963) on $D[0,\infty)$; cf. Whitt (1971, 1972) and Lindvall (1973). Let $\nu_{n,i}$ be the measures induced on $(B[0,\infty), \sigma(J_1))$ by the processes $X_{n,i}$.

Let $f: (S,\pi) \to (B[0,\infty), J_1)$ be defined as follows: $f(s)=x$, where $s=(t_0, t_1, \dots)$ and $x(t)=1$ if $t_{2k} \leq t < t_{2k+1}$ and 0 otherwise, $k=0,1,2,\dots$. It can be shown that f is continuous on S_0. (The function f maps convergent sequences into convergent sequences. Since (R^∞, π) is metrizable, this implies continuity of f; see [5], X.6.3, IX.5.5). Furthermore,

$\nu_{n,i} = \mu_{n,i} f^{-1}$. Since $\mu_{n,i} \overset{\pi}{\Rightarrow} \mu_{0,i}$ and $\mu_{0,i}(S_0) = 1$, it follows by the continuous mapping theorem ([3], p. 29) that $\nu_{n,i} \overset{J_1}{\Rightarrow} \nu_{0,i}$, cf. Whitt (1973).

Now consider "operating processes" associated with the nonrepairable components of the system: For integers $n \geq 0$ and $i = k+1, k+2, \ldots, k+m$ let $Q_{n,i}$ be independent lifetimes with cdf's $F_{n,i}$. Define

$$X_{n,i}(t) = \begin{cases} 1, & t < Q_{n,i} \\ 0, & t \geq Q_{n,i} \end{cases} .$$

Let $\nu_{n,i}$, $i = k+1, k+2, \ldots, k+m$, be the measures induced on $(B[0,\infty), \sigma(J_1))$ by these processes. As above it can be shown that $\nu_{n,i} \overset{J_1}{\Rightarrow} \nu_{0,i}$.

Now consider the process $\underline{X}_n(t) = (X_{n,1}(t), \ldots, X_{n,k+m}(t))$, $t \geq 0$, on the product space $(B^{k+m}[0,\infty), J_1^{k+m})$ of $k+m$ copies of $(B[0,\infty), J_1)$. It follows from the fact that individual components behave independently of one another and consequently from properties of product measures ([3], p. 20) that $\underline{X}_n \overset{D}{\to} \underline{X}_0$ in the J_1^{k+m} topology.

Now consider the coherent structure of the systems Σ_0 and Σ_n. The state of the system is described by a binary $k+m$ dimensional vector, for example, $\underline{x} = (1, 1, 0, 1, 0, \ldots)$. Let $D \subset \{0,1\}^{k+m}$ be the set of states for which the system is "down". Let $U = D^c$ be the set of states for which the system is "up".

Define the real-valued function A_t on $B^{k+m}[0,\infty)$:

$$A_t(\underline{x}(\cdot)) = \int_0^t 1_U\,(\underline{x}(s))ds.$$

$$= \sum_{u \in U} \int_0^t 1_u(\underline{x}(s))ds$$

In $[0,t+1]$ there exist a finite number of transitions; suppose $\underline{x}$ has $n_{\underline{x}}$ transitions in this interval. If $\underline{x}_n \xrightarrow{J_1^{k+m}} \underline{x}$ then, given $\varepsilon>0$, no transition-times of $\underline{x}_n$ in $[0,t+1]$ are perturbed by more than ε from those of $\underline{x}$ for sufficiently large n. This means that each visit to u can be shortened or lengthened by at most 2ε, and since there can be at most $n_{\underline{x}}$ visits to u it follows that

$$\left| \int_0^t 1_u(\underline{x}(s))ds - \int_0^t 1_u(\underline{x}_n(s))ds \right| < 2\varepsilon n_{\underline{x}}$$

for sufficiently large n. Because ε is arbitrary it follows that

$$\lim_{\underline{x}_n \to \underline{x}} \int_0^t 1_u(\underline{x}_n(s))ds = \int_0^t 1_u(\underline{x}(s))ds.$$

Thus A_t is continuous everywhere.

Define the real-valued function L on $B^{k+m}[0,\infty)$:

$$\begin{aligned} L(\underline{x}(\cdot)) &= \inf\{t:\underline{x}(t) \in D\}. \\ &= \min_{d \in D} \inf \{t:\underline{x}(t) = d\} \\ &= \min_{d \in D} L_d(\underline{x}(\cdot)). \end{aligned}$$

We shall show that, for all d, L_d is continuous at all points $\underline{x}(\cdot)$ which have no simultaneous state-transitions by two components. Let $\underline{x}$ be such a point. Suppose $L_d(\underline{x}(\cdot)) = t_{\underline{x}}$. Each component can have only finitely many state-transitions in $[0, t_{\underline{x}}+1]$ and since no simultaneous transitions occur there must be a mimimum distance, $\delta>0$, between two transitions. Now suppose $\underline{x}_n \xrightarrow{J_1^{k+m}} \underline{x}$, then by definition there exist $\lambda_{n,i}$, $i=1, 2, \ldots, k+m$ and N_ε such that $x_{n,i} = x_i \circ \lambda_{n,i}$ and for $n \geq N_\varepsilon$, $|\lambda_{n,i}(s) - s| < \varepsilon \leq \delta/2$ for $0 \leq s \leq t_{\underline{x}} + \delta/2$. Since the transitions of $\underline{x}$ are more than a distance δ apart, it follows that the transitions in $[0, t_{\underline{x}}]$ must be the same for $\underline{x}$ and $\underline{x}_n$, $n \geq N_\varepsilon$; the times of the transitions may be slightly shifted but the sequence of transitions will be identical. Thus $\underline{x}_n$ will first reach d in the interval $[t_{\underline{x}}-\varepsilon,$

$t_{\underline{x}}+\varepsilon]$ and thus $|L_d(\underline{x}_n) - L_d(\underline{x})| < \varepsilon$, for $n \geq N_\varepsilon$. Thus L_d is continuous at $\underline{x}$. This also implies L is continuous at $\underline{x}$, a path with no simultaneous transitions. If $\underline{X}_0$ is the operating process of Σ_0 then L is continuous a.s. relative to the measure induced by $\underline{X}_0$ on $B^{k+m}[0,\infty)$, providing that no simultaneous transitions occur.

Invoking the continuity theorem of the theory of weak convergence ([3], Theorem 5.1) gives, as $n\to\infty$,

$$L_n \overset{D}{=} L(\underline{X}_n(\cdot)) \overset{D}{\to} L(\underline{X}_0(\cdot)) \overset{D}{=} L_0$$

and

$$A_n(t) \overset{D}{=} A_t(\underline{X}_n(\cdot)) \overset{D}{\to} A_t(\underline{X}_0(\cdot)) \overset{D}{=} A_0(t)$$

where " $\overset{D}{=}$ " signifies equivalent probability laws (or distributions). This completes the proof of the Continuity Theorem 2.1.

It is possible to extend Theorem 2.1 to systems with instantaneous failures ($F_{0,i}(0) > 0$) as follows: For integers $n \geq 0$, $1 \leq i \leq k$, define

$$F^*_{n,i} = (F_{n,i} - F_{n,i}(0))/(1 - F_{n,i}(0))$$

$$G^*_{n,i} = \sum_{j=1}^{\infty} F_{n,i}(0)^{j-1}(1-F_{n,i}(0))G^{*j}_{n,i}$$

where G^{*j} is the j-fold convolution. A system with lifetime and repairtime cdf's $F^*_{n,i}$ and $G^*_{n,i}$ will have the same distributions of time until first system failure and system availability as a system with cdf's $F_{n,i}$ and $G_{n,i}$. If $F_{n,i} \overset{D}{\to} F_{0,i}$, $G_{n,i} \overset{D}{\to} G_{0,i}$, <u>and</u> $F_{n,i}(0) \to F_{0,i}(0)$ as $n\to\infty$, then $F^*_{n,i} \overset{D}{\to} F^*_{0,i}$ and $G^*_{n,i} \overset{D}{\to} G^*_{0,i}$ and Theorem 2.1 is applicable.

Barlow and Proschan(1975), chapter 7, describe some other models for maintained systems for which analogous continuity theorems should hold.

Other continuity theorems may be found in [6,7,8,10,15,17,19,20] and the references contained therein. The processes $\underline{X}_n$, $n \geq 0$, in the proof of Theorem 2.1 are actually generalized semi-Markov processes; thus Theorem

2.1 is contained as a special case of continuity theorems proved by Whitt (1976) and Hordijk and Schassberger(1976). However the proof of 2.1 is much simpler than that required for the general processes treated in these papers.

3. Examples with absolutely continuous distributions which have bounded failure rates.

The examples in section 1 were pathological in the sense of having degenerate distributions. In this section the existence of less pathological examples is verified using the continuity theorem (Theorem 2.1). In particular, the degenerate distributions in the preceding examples are replaced by distributions which are absolutely continuous and furthermore whose failure rates are bounded away from 0 and ∞. This will be accomplished by approximating the degenerate distributions by smooth ones.

Lemma 3.1. Let $F_0(t) = 1_{[a,\infty)}(t)$. Define the continuous polygonal failure rate functions $r_{a,n}(t)$,

$$r_{a,n}(t) = \begin{cases} n^{-1}, & t \leq a-n^{-1} \\ \text{linear}, & a-n^{-1} \leq t \leq a+n^{-1} \\ n, & a+n^{-1} \leq t \end{cases}$$

Let $F_n(t) = 1 - \exp(-\int_0^t r_{a,n}(s)ds)$. Then $F_n \overset{D}{\to} F_0$, as $n\to\infty$.

Proof: Follows immediately from definitions.

Now we reconsider example (i) of section 1. Let Σ_0 be a 1-out-of-2 system with distributions given by (1.1) and (1.2). Let Σ_n be a 1-out-of-2 system: component #1 has failure rate function $r_{a_1,n}(\cdot)$ and component #2 has failure rate function $r_{a_2,n}(\cdot)$ (as defined in Lemma 3.1). Let the repair distributions of Σ_n be identical to those of Σ_0. Then the Continuity Theorem 2.1 and Lemma 3.1 imply $L_n \overset{D}{\to} L_0$, as $n \to \infty$.

It is easy to see that F_{L_0} is continuous at $t = 11$ and 22. Thus $\lim_{n\to\infty} F_{L_n}(t) = F_{L_0}(t)$ for $t = 11, 22$. This, plus the fact that $\overline{F}_{L_0}(22) > [\overline{F}_{L_0}(11)]^2$, implies that there exists an N such that for $n \geq N$, $\overline{F}_{L_n}(22) > [\overline{F}_{L_n}(11)]^2$. Thus there exists a 1-out-of-2 system, Σ_N, whose component lifetimes are IFR (with rate bounded away from 0 and ∞) and exponential repair times such that system lifetime is not NBU.

In light of this example for bounded failure rates, there may not exist any useful general restrictions on component lifetime distributions which guarentee NBU system lifetime, except the assumption of Barlow and Proschan (1976) of exponential lifetime distributions.

A similar analysis based on example (ii) of section 1 verifies the existence of a similar 1-out-of-2 system with nondegenerate IFR component lifetimes and exponential repair times for which increased repair rates lead to lower system reliability and availability.

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